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Heterogeneous objects representation for Additive Manufacturing: a review

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Abstract

Recent advances in additive manufacturing technologies demand for extremely customized, complex shape and multi-fold functional products. Heterogeneous objects, such as functionally graded materials, represent an attractive solution for researchers and industries in many application fields. Combining geometric modelling and material assignment in a definitive and accessible CAD tool is still a challenge. In this review the key aspects of heterogeneous object representation related to additive manufacturing processes are reported. After the presentation of the various methodologies for geometric modelling found in the literature, additive manufacturing applications for heterogeneous objects are summarized.

Keywords: Geometric modeling; Computational geometry; Additive manufacturing; CAD; FGM; Heterogeneous objects

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Introduction

Thanks to the exploitation of design freedom guaranteed by additive manufacturing (AM) technologies, heterogeneous object modelling and production is receiving a renewed interest. These lead to a paradigm shift that is taking place in industry, from a shape-centred approach to a functional requirement approach [1]. Nowadays, AM processes can create heterogeneous objects, but the lack of suitable geometric modelling and material

representation techniques for this type of objects limits the ability of product development [2,3]. Current CAD tools and workflows have been designed to represent homogeneous objects suitable for traditional manufacturing technologies, modelling shapes by their boundaries without information about the material distribution inside the part.

We experienced heterogeneous objects much more than homogeneous ones. Manmade objects are usually almost constant in their internal material distribution and they can be modelled as homogeneous solids. In contrast,

natural objects are rarely homogeneous, fitting together functions, shapes and materials. Nature reveals every day optimized heterogeneous objects such as animal tissues (e.g. human bones), plant structures (e.g. wood) and geological materials (e.g. soil and rocks) [2,4,5]. For this reason, also related to biomimicry design approach, the interest in heterogeneous objects has increased exponentially [6]. Heterogeneous object modelling is not a novelty in computational design, as it represented the natural evolution of homogeneous object modelling [7,8]. A heterogeneous object is referred to a solid component consisting in more than two attributes distributed discontinuously or continuously inside geometry boundaries [9,10]. If a discontinuous change in attributes distribution, i.e. material distribution, generates distinct regions separated by distinct interfaces in the solid, it is called a composite (Figure 1a) [10]. On the other hand, if the continuous variation of an attribute produces gradient in material distribution, it is often referred to as functionally graded material (FGM) [10]. In the simplest FGM, two different materials change gradually from one to the other, as schematically shown in figure 1b. In addition, distribution can be even random or irregular [4,5,11]. Sometimes, authors use FGM and multi-material terms interchangeably [12] while they use composite for the discontinuous change. The effective material properties such as Young's modulus, Poisson's ratio, density, thermal conductivity, and thermal expansion can be determined by several rules such as the mixture, the three-phase model by Frohlich and Sack, the self-consistent scheme, the Mori-Tanaka technique, and the mean field approach [13].

FGM are now a consolidate argument in scientific research. This is demonstrated by the growing number of publications in the literature. Indeed, according to Scopus database, the number of papers containing the keyword "functionally graded materials" increased from 153 in 2000 to 924 in 2018. For all the reasons above, a shared systematic

design methodology is urgently required, specifically integrated with additive manufacturing methods and tools [1,4,12]. This means to elaborate an overall design approach, in order to consider object multi-fold functions, shape and material distribution in specific environmental and boundary conditions. As a first step in this direction, in this work firstly a taxonomy of geometric modelling approaches for heterogeneous objects is proposed, and then, the opportunities highlighted in literature for manufacturing heterogeneous objects by AM technologies are presented.

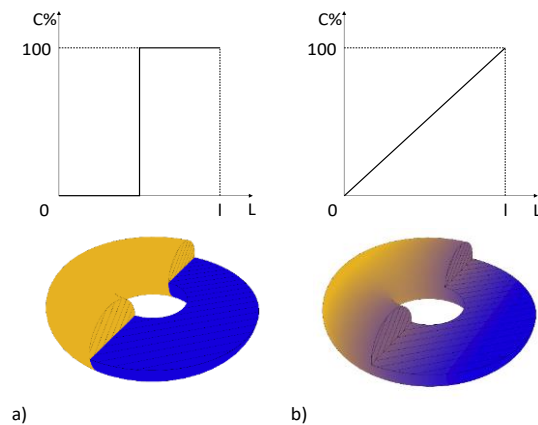


Figure 1: A heterogeneous model of a torus, (a) with discrete material distribution and (b) with graded material distribution. Yellow and blue colors represent two different materials. In the length axis (L), l represents the length of the object. C% is the percentage of blue material: when C%=0 the object is totally made of yellow material, if C%=100 the object is blue.

Heterogeneous object modelling and representation

Current CAD software are designed to operate with homogeneous solids and not with heterogeneous ones, because the modelling objective until now was to describe mainly geometrical information [14]. Many models exist for representing the outer shape of an object, but modelling the inner composition is still a challenge [1,15,16]. Since heterogeneous

objects modelling is not trivial, because includes more than the shape representation, there are different approaches to model the volumetric property distribution in a solid [4,12,17], some of those take advantages from the analogies with other disciplines such as geosciences or colour representation [18].

In general, heterogeneous object geometric modelling requires the connection of properties information, such as material, to the solid model. This procedure includes two concurrent phases: shape modelling and material modelling or properties assignment [4,12,17,19,20]. Geometric modelling is concerned with both shape representations of the objects and properties assignment that is targeted at defining property distribution and composition over the geometric domain [4]. The procedures for heterogeneous object modelling can intuitively be sequential or parallel, that is whether the shape is modelled before material distribution, or the geometry and the material are defined simultaneously, respectively. As almost all the commercial CAD software packages can only create geometric models, now sequential process is the simpler strategy to model FGM object [12]. Alternatively, as Boddeti et al. [21] or Garland et al. [15] proposed, it is possible to simultaneously define topology and material distribution by an original algorithm based on topology and material gradient optimization within a single part. Zhang et al. [12] refers to boundary modelling and property assignment in terms of attributes. Indeed, shape attribute is usually already defined, while other properties or attributes that may differ from the material, can be defined as well. FGM fundamental attributes/properties are geometry and material. Other attributes/properties are for example microstructure [22-26], tolerances and operating conditions, which could also be included in a complete model description [2,8,14,27]. In order to define the better distribution of material, new design and optimization approaches are needed. For instance, Tornabene et al. [28,29] proposed a method for designing shells with graded

composition between ceramic and metal along the lamina thickness, to optimize frequency and static deflection.

Geometric modelling approaches

Geometric modelling approaches for solids can be classified in 3 main classes (Figure 2): boundary representation (B-Rep), volume representation (V-Rep) and constructive solid geometry (CSG) [30,31]. In B-rep solids are described in terms of connected surfaces or faces representing the surface of an object. V-rep allows the description of both the surface and the internal portion of an object, and consequently the representation of the internal properties' distribution and not only of the boundaries. CSG is a description of a solid geometry through sequential logical operations (Boolean), starting from simple primitive geometries that could be homogeneous or heterogeneous. In the second case, Boolean operations should be redefined to manage heterogeneous primitives [30,32]. In order to visualize and manufacture the model, depending on the primitives, it is possible to shift to a B-rep or a V-rep.

Both B-rep and V-rep can be represented by discrete models or by functions. In the case of discrete V-rep, spatial decomposition can be reached by voxel, octree and polyhedral mesh. Typical discrete representation can be found in computed tomography (CT) image representation, in voxel form, or in finite element (FE) method, as polyhedral mesh. Beside discrete representation methods, bi-variate or tri-variate parametric functions are often used in CAD software for intuitively modify shape by moving control points. Usually, bi-variate parametric functions are used for surface representation, while tri-variate are implemented for morphing objects. Implicit and explicit functions can be implemented in CAD software and can be useful in the representation of particular shapes such as minimal surfaces, but do not allow an easy modification of the shape as in the case of parametric functions.

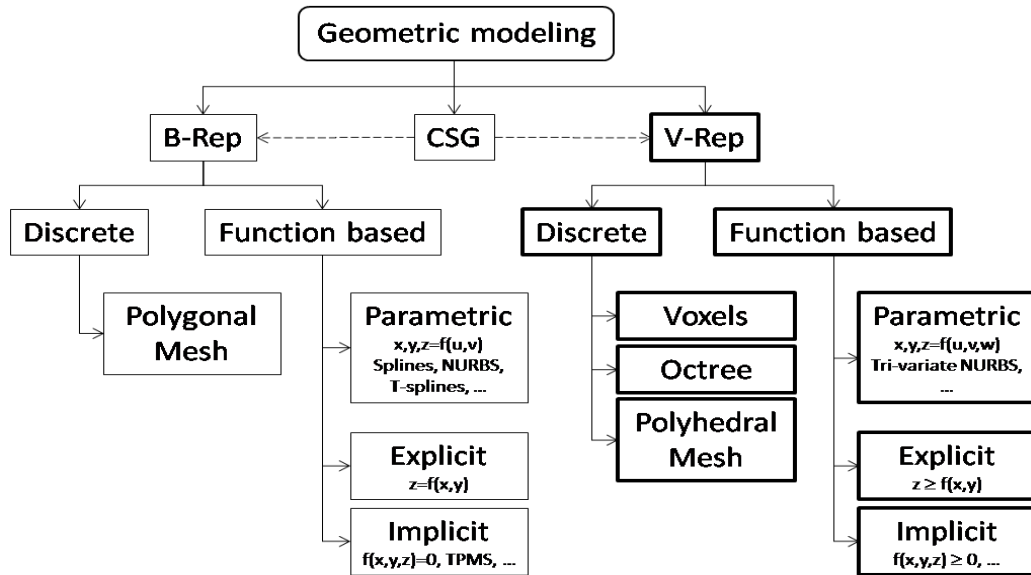


Figure 2: Geometric modelling approaches.

Discrete V-rep representation

Discrete volumetric models are often classified as evaluated models or representations [4], since the volumetric information stored in the model is directly available for further application, such as numerical analysis and simulation [11]. The easiest way to discretize a volume is to subdivide it in small and equal cubes (i.e. voxel). Octrees is a partition of a three-dimensional space by recursively subdividing a cube into eight cubes; each cube can be inside, outside or in the boundary of an object; for each cube in the boundary the subdivision is repeated until the desired resolution is obtained. The space decomposition can be reached by other approaches that can better follow the external shape or the internal characteristics, using polyhedra. In this case, it is possible to adopt larger elements where the variation in shape or characteristics is low and vice versa. The discrete representation schemes are based on different data structures: for instance, voxel based methods rely on the distribution of the elements inside a tree-dimensional matrix or a vector, octree is based on a tree, while for

polyhedral mesh it is necessary to define a list of vertex coordinates, a list of polyhedra, a list of faces, a list of edges and the reciprocal connections [30,33,34].

Function based V-rep representation

The most common way to define a geometry in a CAD environment is based on parametric functions. In the volumetric case, these functions map a domain of the parameter space (u,v,w) in the design space (x,y,z). Function adopted for mapping the (u,v,w) space in the (x,y,z), can be simple polynomials, but for an easier geometry management, control points were introduced together with basis function in the Bézier representation. In this formulation, moving a point, the whole geometry is modified. To overcome this limitation, B-splines and NURBS were introduced, where rational non uniform basis functions are recursively defined. Using NURBS, with a single equation, any type of geometry can be obtained. Differently, using explicit or implicit functions, each geometry is represented by a different equation, making difficult modeling complex geometries [33].

Properties assignment approaches

For heterogeneous objects, in particular for FGM, the volumetric property information consists in material data, and it can be divided in two branches: material composition and distribution [12]. Composition at each point of the volume is identified by a vector m . This represents a complication, because material information adds dimensions to the model. Distribution, that model the variation of m in the volume, can be described in various way and can be classified in three main classes [12]. The first ones is the extension of conventional geometric modelling approaches in order to consider material representation, i.e. the material description is dependent to the geometry approach used. This is called geometric model-based. The second class contains schemes wherein the material assignment is based on other geometry information, e.g. coordinate system-based. In the last class, special control feature-based schemes, also referred to as material primitives' features, are used to describe material distribution in FGM objects [11]. Other classifications are possible and not in contrast with the previous one [4]. Anyway, the material assignment needs a geometric support. Conceptually, next to a build space where the geometry is defined with any of the approaches previously described, a geometric model for material distribution is added. The basic concept of any heterogeneous modelling method is to define a function associating the material to all the points in the geometric model [20].

Geometric model-based

In geometric model-based, geometric modelling approaches are utilized as the basis for modelling the material attribute. In this case, material distribution is geometric representation dependent. In discrete V-rep, at each voxel, polyhedron or vertex within a boundary, a material composition is assigned [30]. These models permit complex FGM modelling with great accuracy that is directly

related to the domain resolution, but computational and memory costs can be high [4,30]. For example, voxels approach can be improved by bringing together adjacent voxel with the same properties, making the spatial-occupancy enumeration more efficient such as in octree encoding [13]. Note that the material distribution inside a voxel is not necessarily homogeneous, for instance, Bernstein polynomials or tri-linear functions have been used to represent interpolated material distributions when the material composition is assigned to a vertex [4]. Extensions to pure voxel representation has been proposed by several authors, such as Blouin et al. [35]. A more flexible approach is based on polyhedral mesh. In polyhedral mesh, objects are described with a set of adjacent polyhedra, each represented by a list of vertices. The vertices store their geometric position as well as the material composition, and can be mathematically described as:

$$\text{heterogeneous object} = \{V_i\} = \{T_1, T_2, \dots, T_n\}$$

$$T_k = \{G(v_{k1}, v_{k2}, \dots, v_{km}), M(v_{k1}, v_{k2}, \dots, v_{km})\}, \\ 1 < k < n$$

$$v_{ki} = (x_i, y_i, z_i, m_i)$$

where T_k denotes a representative polyhedron, G and M denote the geometry and material distribution of T_k , v_{ki} is a representative vertex of T_k that stores the coordinates and punctual material information m_i and n is the number of polyhedrons that compose the whole heterogeneous object. The function M is an interpolation function used for defining the material distribution inside each polyhedron.

Contrary to space subdivisions, function-based representation utilizes exact geometric data representations, such as B-Rep and f-Rep [36], and rigorous functions (explicit, implicit, parametric) to represent the material distributions [4]. For implicit and explicit functions, it is very challenging to work on different levels of inequalities to manage the distribution of different materials. For example, with an implicit function based strategy it is possible to set a property at each k_i value (a

surface) of the $f(x,y,z)=k_i$, but it has some drawbacks. Instead, parametric functions, such as tri-variate NURBS, show several advantages and the approach can be extended in FGM representation [30,37,38].

Coordinate system-based and control features-based

Material distribution can be defined based on geometrical features which differ from the shape of the object. For instance, the definition of properties or material can be referred to the coordinate system and independent from the boundary of an FGM. This is referred as coordinate system-based [12,17]. The distribution is defined with respect to a Cartesian, cylindrical or spherical coordinates system with linear or non-linear and discrete or continuous functions. If the material is assigned to a set of equidistant points, the supporting geometric model is equivalent to a voxel model. Doubrovski et al. [39] proposed a methodology based on voxelization modelling in which the resolution is set equal to the additive manufacturing process. Another example appears when the distribution is controlled by features such as points, curves or surfaces, referred as material features. Material composition at any point in the space model is derived from these control features and distance-based weighting functions. As stated in [4,11] this approach seems to be more intuitive from a user experience point of view. Bidarra et al. [27] defined a feature as a representation of the shape aspect of a product that is mappable to generic shape and functionally significant for the product. Also, features may be entities that are not otherwise present in the shape model. These new reference entities may be point (0-D), line/curve (1-D), or plane/surface (2-D). The material distribution function can be polynomial, exponential or harmonic functions of the distance from material reference entities [1,4,5,12]. Recently developed voxel-based modeling engine called Monolith [40] permits to handle spatial variations directly in material properties, using different approaches to assign

the properties to objects such as geometric model-based, coordinate system-based and control features-based. These voxel-based representation fits perfectly within a new class of 3D printers which have multiple print heads capable of depositing different types material, such as resin, within a single build volume.

Additive Manufacturing applications

Manufacturing techniques play a critical role in achieving the designed composition and thus the demanded properties of heterogeneous object for specific applications. In particular, FGMs have found applications in various fields, such as aerospace, mechanical, electrical, thermal, optical, biomedical and geophysical [1,4,12,41]. One way to categorize these techniques is based on the type of FGM objects manufactured by them. Mahamood et al. [42] classified the FGM objects into two groups: thin and bulk FGM. Thin FGM is usually in the form of surface coatings, while manufacturing techniques for bulk FGM are powder metallurgy, centrifugal method, and AM [12]. Not all the current AM technologies, classified in ISO/ASTM 52900 [43], are now used for FGM realization. The main AM techniques reported in literature are presented in figure 3: vat photopolymerization (Stereolithography – SLA), material extrusion (Fused Deposition Modelling – FDM), powder bed fusion (Selective Laser Sintering/Melting – SLS/SLM), direct energy deposition (Laser Engineered Net Shaping – LENS) and material jetting (polyjet) [12].

SLA has attractive attributes of creating objects with a high-quality surface finish, dimensional accuracy, and a variety of material options. The material distribution is homogeneous in a layer, but changes along the build direction. It is challenging to obtain heterogeneous material compositions within intralayer. However, there is a possibility of printing functionally graded material with SLA. As shown by Huang et al. [44], a mask-image-projection-based Stereolithography is proposed to build objects with multiple materials.

FDM produces parts by extruding filaments of molten thermoplastics material through heated nozzles. After extrusion from the nozzle in a desired pattern, the material solidifies to form the object. There are large varieties of materials that can be used in FDM process. FDM devices with multiple nozzles allow the construction of heterogeneous objects with discrete material distribution. On the other hand, it is possible to put different materials in the same nozzles having the potential of manufacturing functionally graded material objects as long as the machine system allows for an arbitrary mixture of different filament materials. For example, Leu et al. [45] developed a triple extruder mechanism, which can control the filaments extrusion for desired composition gradients. In the same way Garland et al. [15] used an off the shelf FDM 3D printer to produce FGM object. The printer is equipped with a nozzle that can extrude two mixed materials at once. By controlling the rate at which the two filaments are pulled into the melt chamber, FGM objects can be printed, i.e. colours or compositions changed. Khalil et al. [46] showed the possibility of constructing heterogeneous tissue with FDM process in medical applications. Their system is based on a setup with four different nozzles.

LENS and SLS-SLM are promising technologies for fabricating FGM metal parts with excellent strength, accuracy (50-100 μm), and surface roughness (<10 μm), depending upon the machine type, materials and geometry of the products. Both LENS and SLS-SLM use

powders as construction unit, but the former in blown-powder while the latter in a powder-bed technique. By controlling the composition ratio of different material powders, they have the potential of producing FGM objects. LENS is mainly used for iron-, titanium-, and nickel-based alloys. Other examples of FGM parts are functionally graded tungsten carbide and tool steel parts, alloys and ceramic parts by SLM, TiC and Ti composite by LENS, and Nykon-11 and silica nanocomposites by SLS [9,12]. Stratasys PolyJet 3D printing technology jets layers of curable liquid photopolymer onto a build tray and the gradient profile is thus continuous. One application of this technology for a graded prosthesis production is proposed in [39], where a PolyJet printer permits the elimination of slicing and path planning by bitmap images. Other researchers have demonstrated workflows for modelling and fabricating material compositions with target visual properties and desired deformation behaviour [39]. Another example is given by Connex 3 by Stratasys [47]: it offers the ability to create objects by jetting material droplets in a predefined pattern from designated microscale inkjet printing nozzles. With a three-base colour system, the material droplets have a wide colour range option from 20 palettes, each one providing several colours. The process requires a specific range of viscosity and curing temperature of the jetted liquid. This limits the type of material that can be used in this process.

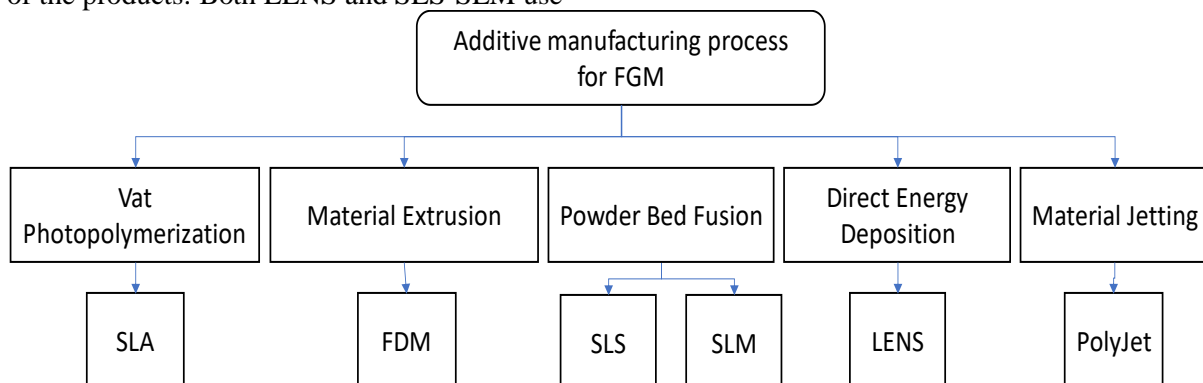


Figure 3: Additive Manufacturing processes currently used for FGM objects, classified using ISO/ASTM 52900.

Conclusions

AM can effectively build heterogeneous object, but research effort must be addressed to improve design and representation methods. In order to support these trends, this research suggested a classification of the possible approaches for volumetric modelling. Different approaches to assign properties of heterogeneous objects were described, defining a fundamental step for any design approach able to optimize the material combination and distribution on a design space. Moreover, the paper presents the major AM technologies useful for manufacturing FGMs.

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