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A comprehensive study of super mean labelling in three star graph

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Abstract

In this paper, we prove that if any three star graph $G = K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$, $\ell \le m \le n$ admits Sub Super Mean Labeling with $2 \le \ell, 2 \le (n-m)+\ell$, where $g = \left\lceil \frac{n+\ell-m}{2} \right\rceil$, then G is a g RO graph. let G be a (p, q) graph and $f : V(G) \to \{1, 2, 3, \ldots, p+q\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$ is odd. Then f is called sub super mean labeling if $f(V) \cup \{f^*(e) : e \in E(G)\} \subset \{1, 2, 3, \ldots, p+q\}$. A graph that admits a sub super mean labeling is called a sub super mean graph.
Keywords: SSMG; SSML(V_4,E_2) ; FEIO; SEIO; FOIO**;** Star 2010 Mathematical subject classification Number: 05C78

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Introduction

Here our discussion is on a three star graph which is finite, simple and undirected one. For notations and terminology, we follow [1]. The vertex set and the edge set of a graph G are denoted by V (G) and E (G) respectively. So $G(V, E)$ is a graph with p vertices and q edges. Much work is done by researchers on Mean labeling and Super mean labeling of graphs applying them on a variety of graphs [2-7,9,10]. Motivated by these work the concept of SSML(V_4,E_2) on a two star graph was introduced by Uma Maheshwari et al [8]. After a discussion on SSML(V_4,E_2) on a two star graphs, we were inspired to apply SSML(V_4,E_2) on three star graphs and hence this paper**.** Here

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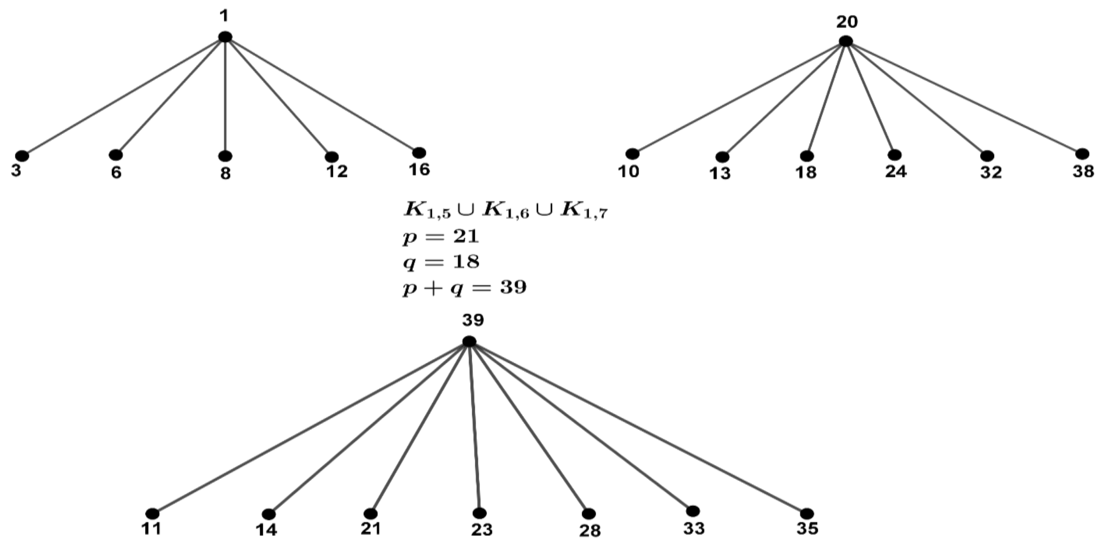
the word sub is used thereby allowing omissions which lead to repetitions and a special case $SSML(V_4, E_2)$. A study on omissions and repetitions on a three star graphs is done which admits $SSML(V_4, E_2)$ and the formula is obtained in terms of ℓ, m and n in this paper.

Results and Discussions

Definition 2.1: The three star graph is the disjoint union of $K_{1,\ell} \cup K_{1,m}$ and $K_{1,n}$ It is denoted by $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$.

Definition 2.1: Super Mean Labeling

Let G be a (p, q) graph and $f: V(G) \to \{1, 2, 3, \ldots, p+q\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$ is odd. Then f is called super mean labeling if $f(V) \cup \{f^*(e) : e \in E(G)\} = \{1, 2, 3, \ldots, p+q\}$. A graph that admits a sub super mean labeling is called a super mean graph.



Definition 2.2: Sub Super Mean Labeling (Ssml)

Let G be a (p, q) graph and $f: V(G) \to \{1, 2, 3, \ldots, p+q\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$ is odd. Then f is called sub super mean labeling if $f(V) \cup \{f^*(e) : e \in E(G)\} \subset \{1, 2, 3, \ldots, p+q\}$.
A graph that admits a sub super mean labeling is called a sub super mean graph.

Definition 2.3: Sub Super Mean Labeling Ssml (V_4,E_2)

A specific SSML is defined on a three star graph where the numbers assigned to the pendant vertices differ by four and the corresponding edge values differ by two, almost everywhere and so it is named as $SSML(V_4, E_2)$ aptly.

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Definition 2.4: $(g-1),(g)$ and $(g+1)$ **RO Sub Super Mean Graph (SSMG)**

If the number of repetitions = number of omissions $=(g-1),(g)$ or $(g+1)$ with respect to SSML(V_4,E_2) then the three star graph G is called $(g-1),(g)$ or $(g+1)$ RO sub super mean graph where $g=\left[\frac{n+\ell-m}{2}\right]$.

Definition 2.5: INTEGRAL PART

For any real number x, the greatest integer $\leq x$ is denoted by [x], is the integral part of x.
If x is an integer [x] = x.
If x is not an integer [x] < x.

Definition 2.6: First even integer omitted (FEIO)

While assigning numbers to the pendant vertices using SSML(V_4,E_2) an even integer gets omitted moving from $K_{1,\ell}$ to $K_{1,m}$. This even integer is the first even integer omitted and is denoted by FEIO = $2\ell+2$.

Definition 2.7: Second even integer omitted (SEIO)

While assigning numbers to the pendant vertices using SSML(V_4,E_2) an even integer gets omitted in moving from $K_{1,m}$ to $K_{1,n}$. This even integer is the second even integer omitted and is denoted by SEIO = $2\ell+2m+4$.

Definition 2.8: First odd integer omitted (FOIO)

While assigning numbers to the pendant vertices using SSML (V_4,E_2) an odd integer gets omitted in moving from $K_{1,m}$ to $K_{1,n}$. This odd integer is the first odd integer omitted and is denoted by FOIO = $4\ell+7$.

2.9: The discussion of SSML(V_4,E_2) for $G=K_{1,\ell}\cup K_{1,m}\cup K_{1,n}$ for $2\leq\ell, 2\leq(n-m)+\ell$ is provided below which is required for the theorem.

Let $G=K_{1,\ell}\cup K_{1,m}\cup K_{1,n}$

$p=\ell+m+n+3$

$q=\ell+m+n$

$p+q=2\ell+2m+2n+3$

In the first copy of $G=K_{1,\ell}\cup K_{1,m}\cup K_{1,n}$ the labeling is done as follows:

$f(u)=1,$

$f(u_1)=3,$

$f(u_i)=3+4(i-1),\ 2\leq i\leq\ell$

$f(u_\ell)=3+4(\ell-1)=4\ell-1,$

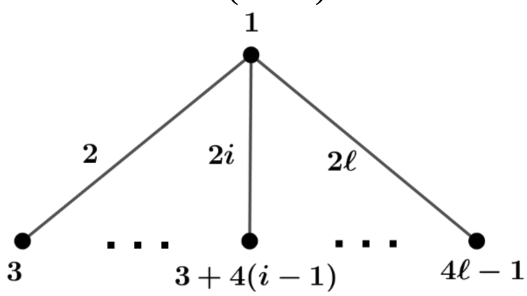
The corresponding edge labeling is given by

$f^*(uu_1)=2,$

$f^*(uu_i)=2i,\ 2\leq i\leq\ell$

$f^*(uu_\ell)=2\ell,$

Note that FEIO is $(2\ell+2)$



In the second copy of $G=K_{1,\ell}\cup K_{1,m}\cup K_{1,n}$ the labeling is done as follows:
Define

$f(v)=f(u_\ell)+4=(4\ell-1)+4=4\ell+3,$

$f(v_1)=5,$

$f(v_j)=5+4(j-1),\ 2\leq j\leq m$

$f(v_m)=5+4(m-1)=4m+1,$

The corresponding edge labeling is given by

$f^*(vv_1)=2\ell+4$

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$$f^*(vv_j) = \frac{(4\ell+3)+5+4(j-1)}{2}$$

$$= \frac{4\ell+4j+4}{2}$$

$$= 2\ell+2j+2, \quad 2 \le j \le m$$

$$f^*(vv_m) = 2\ell+2m+2$$

Note that SEIO is $(2\ell+2m+4)$

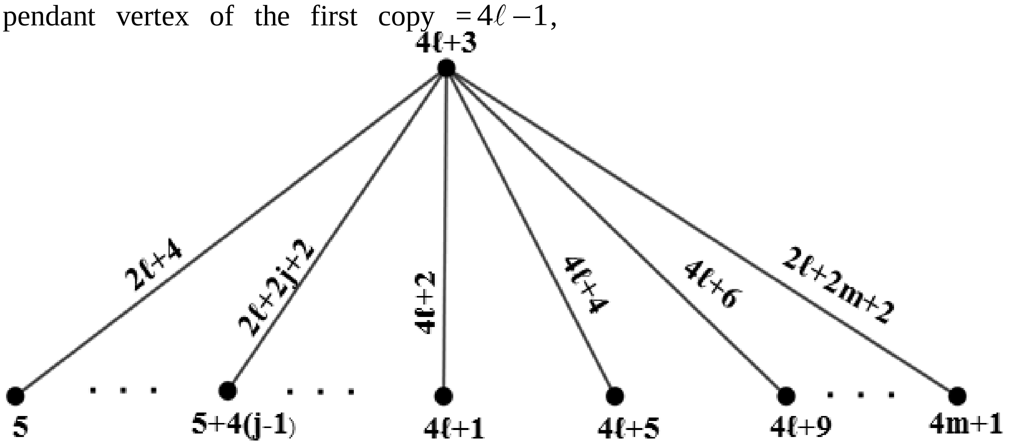
The pendant vertices of the first and second copies differ by two correspondingly, the ℓ^{th} pendant vertex of the first copy $= 4\ell-1$, therefore the ℓ^{th} pendant vertex of the second copy $= 4\ell+1$.

The $\ell^{th}, (\ell+1)^{st}, (\ell+2)^{nd} \ldots$ pendant vertices of the second copy are $4\ell+1, 4\ell+5, 4\ell+9, \ldots$.

Up to the ℓ^{th} place, no odd integers are omitted and $4\ell+3$ is allotted to f(v).

So, FOIO is $4\ell+7$, this should be the first pendent vertex of the third copy of G.

Note that SEIO is $(2\ell+2m+4)$



In the third copy of $G = K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ the labeling is done as follows:

Define

$$f(w) = f(v_m) + 4 = (4m+1) + 4 = 4m+5$$

$$f(w_1) = 4\ell+7,$$

$$f(w_k) = (4\ell+7) + 4(k-1), \quad 2 \le k \le K$$

To find K

As $p+q = 2\ell+2m+2n+3$,

$$(4\ell+7)+4(k-1) \le 2\ell+2m+2n+3$$

$$4k \le 2m+2n-2\ell$$

$$k \le \left[\frac{m+n-\ell}{2}\right]$$

$$K = \left[\frac{m+n-\ell}{2}\right]$$

$$f(w_{K+1}) = 2\ell+2m+2n+3,$$

$f(w_{K+2})$ to $f(w_n)$ are allotted all the remaining odd integers, FEIO and SEIO if required.

The corresponding edge labeling is given by

$$f^*(ww_1) = 2\ell+2m+6$$

$$f^*(ww_k) = \frac{(4m+5)+(4\ell+7)+4(k-1)}{2}$$

$$= 2\ell+2m+2k+4, \quad 2 \le k \le K$$

The edge values $f^*(ww_{K+1})$ to $f^*(ww_n)$ assume only odd integral values (proof given below) whether the end vertices are odd and FEIO or SEIO.

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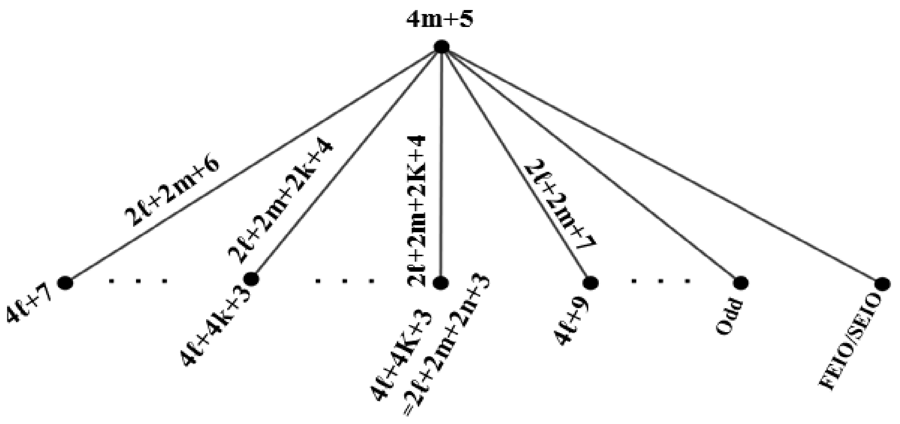

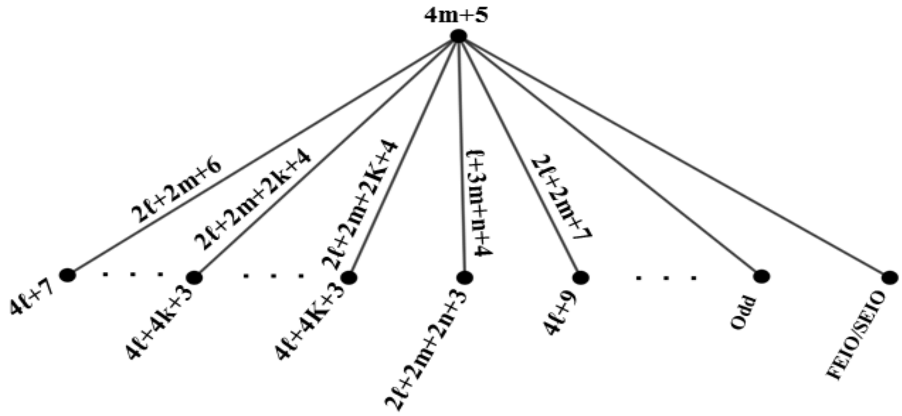

Theorem 2.10:

If $G = K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ admits $SSML(V_4, E_2)$ with $2 \le \ell,\ 2 \le (n-m)+\ell,\ g = \left[\frac{n+\ell-m}{2}\right]$, then

(i) G is a g RO SSMG if any one of FEIO or SEIO is assigned to an end vertex.

(ii) G is a $(g-1)$ RO SSMG if both FEIO and SEIO are assigned to the end vertices.

(iii) G is a $(g+1)$ RO SSMG if both FEIO and SEIO are not assigned to the end vertices

Claim (i) If a is any odd integer, b, c, d and e are consecutive odd integers such that $b < c < d < e$ and if $\frac{a+b}{2}$ is an even integer then $\frac{a+d}{2}$ is also an even integer and $\frac{a+c}{2}$ and $\frac{a+e}{2}$ are odd integers, where $|b-c| = 2$, $|b-d| = 4$, $|c-d| = 2$, $|c-e| = 4$,

$$\frac{a+b}{2} = 2k$$

$$a = 4k - b$$

$$\frac{a+d}{2} = \frac{4k-b+d}{2} = \frac{4k+4}{2} = 2k+2\,(even)$$

$$\frac{a+c}{2} = \frac{4k-b+c}{2} = \frac{4k+2}{2} = 2k+1\,(odd)$$

$$\frac{a+e}{2} = \frac{4k-b+e}{2} = \frac{4k+6}{2} = 2k+3\,(odd)$$

Claim (ii) The edge values from the $(K+1)^{st}$ position of the third copy of $G = K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ are odd integers**.**

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Here $(K+1)^{st}$ position of the pendant vertex of the third copy is an odd integer $2\ell+2m+2n+3$.

The odd integer assigned to the K^{th} pendant vertex is $4\ell+4K+3$.
The difference between the numbers allotted to the pendant vertices of the K^{th} position is $(4\ell+4K+3)$ and $(K+1)^{th}$ position is $(2\ell+2m+2n+3)$ cannot be one or three as both are odd, and it cannot be four also.
If the difference is four, $2\ell+2m+2n+3$ corresponds to the K^{th} pendant vertex.
Therefore, the difference is two.

So, $\frac{f(w)+(2\ell+2m+2n+3)}{2}$ is an odd integer from claim (i)

Therefore, the edge value corresponding to the $(K+1)^{st}$ position is an odd integer.

Also, from $(K+2)^{nd}$ position onwards, when the pendant vertices assume odd integers, their edge values must be odd integers from claim (i).

Claim (iii): FEIO is an end vertex for the third copy of $G=K_{1,\ell}\cup K_{1,m}\cup K_{1,n}$ if $\ell+2m+4$ is odd.
Here FEIO is $2\ell+2$

$f(w)=4m+5$

As $2\ell+2<4m+5$ and $2\ell+2<4\ell+7$, $2\ell+2$ cannot be the average of $4\ell+7$ and $4m+5$, so $2\ell+2$ cannot be an edge value at all.

To check, if $2\ell+2$ is a pendent vertex of the third copy of G.

$$\frac{(2\ell+2)+(4m+5)}{2}=\ell+2m+4>2\ell+4$$
$$=2m>\ell \quad \text{(true) it can be an end vertex,}$$

check if $\ell+2m+4\le 2\ell+2m+2$,

$$2\le\ell,$$

If $\ell+2m+4$ is even, it occurs already as an edge value in the second copy of G for $2\ell+4<\ell+2m+4<2\ell+2m+2$.

FEIO is rejected if $\ell+2m+4$ is even, due to the repetition of edge values $\ell+2m+4$ occurring in the second copy.
FEIO is rejected if $\ell+2m+4$ is even.
Therefore FEIO is accepted as end vertex if $\ell+2m+4$ is odd.

Claim (iv): SEIO is an end vertex for the third copy of $G=K_{1,\ell}\cup K_{1,m}\cup K_{1,n}$
if $\ell+3m+5$ is odd.
Here SEIO is $2\ell+2m+4$
$f(w)=4m+5$
As $\ell\le m$, $2\ell+2m+4<4m+5$,

The least edge value of the third copy of G is $2\ell+2m+6>2\ell+2m+4$.
So, SEIO cannot be an edge value for the third copy.
To check, if $2\ell+2m+4$ is a pendent vertex of the third copy of G,

$$\frac{(2\ell+2m+4)+(4m+5)}{2}=\ell+3m+5$$

To check $\ell+3m+5\ \ge\ 2\ell+2m+6$

$$m\ge\ell+1$$

Which is not possible, as ℓ, m, n are taken in general.
So, $\ell+3m+5<2\ell+2m+6$
If $\ell+3m+5$ is even with ℓ even and $\ell=m$, $\ell+3m$ is even and so $\ell+3m+5$ is odd.

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If $\ell+3m+5$ is even with 3m even, and if $\ell=m$ it implies ℓ is even which is the same as above.
If $(3m+5)$ is even then $\ell+3m+5$ is even only if ℓ is even.
So, we conclude that $\ell+3m+5$ is not even in general.

So, $\ell+3m+5$ must be odd if SEIO is assigned an end vertex.
Claim (v): The result of the theorem is valid only if $2\le\ell$ and $2\le(n-m)+\ell$.
The edge value corresponding to the K^{th} position is

$$\frac{(4m+5)+(4\ell+4K+3)}{2}$$

$$\frac{(4m+5)+(4\ell+4\left[\frac{m+n-\ell}{2}\right]+3)}{2}$$

$$\frac{(4m+5)+4\ell+(2n+2m-2\ell)+3}{2}$$

$$\frac{2\ell+6m+2n+8}{2}$$

$(\ell+3m+n+4)\le 2\ell+2m+2n+2$ $(2\ell+2m+2n+2)$ is the large even integer available

$$m+2\le\ell+n$$

$$2\le(n+\ell)-m$$

$$2\le(n-m)+\ell$$

Proof

Now a counting scheme is provided for the number of odd and even integers assigned and omitted.
$p+q=2\ell+2m+2n+3$ is an odd integer. So, number of odd integers is one more than the number of even integers.
$p+q=(\ell+m+n+2)+(\ell+m+n+1)$
The total number of odd integers is $(\ell+m+n+2)$ and the total number of even integers is $(\ell+m+n+1)$. $\rightarrow(a)$
The labeling of the first copy and the second copy are filled up for the edge values and pendant vertices. So concentrate only on the third copy.
In the third copy of $G=K_{1,\ell}\cup K_{1,m}\cup K_{1,n}$ the labeling is done as follows:

Define
$f(w)=f(v_m)+4=(4m+1)+4=4m+5$
$f(w_1)=4\ell+7,$
$f(w_k)=(4\ell+7)+4(k-1),\ 2\le k\le K$
The number of odd integers used up to the K^{th} position is
$=(1+\ell)+(1+m)+(1+K)$
$=3+\ell+m+K$
$=3+(\ell+m)+\left[\frac{m+n-\ell}{2}\right]$

$$=\left[\frac{6+\ell+3m+n}{2}\right] \rightarrow(b)$$

The number of odd integers yet to be assigned in the rest of the end positions from (K+1) to n is

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$$=(\ell+m+n+2)-\left[\frac{6+\ell+3m+n}{2}\right]$$

$$=\left[\frac{\ell-m+n-2}{2}\right] \qquad \rightarrow (c)$$

The edge values and the pendent vertices yet to be allotted is in number,

$$2(n-K)=2\left\{n-\left[\frac{m+n-\ell}{2}\right]\right\}$$

$$=[n-m+\ell] \qquad \rightarrow (d)$$

As $\left[\frac{\ell-m+n-2}{2}\right]<(n-K)$, is true, for, on substitution of K on the RHS we get $\left[\frac{\ell-m+n-2}{2}\right]<\left[\frac{n-m+\ell}{2}\right]$, which is true.

So some pendent vertices may have to assume even integers.

Now we count the number of even integers to be assigned.

In the first copy ℓ even integers, in the second copy m even integers, and in the third copy K even integers are used for edge values.

Total number of even integers available = $\ell+m+n+1$

The difference is found out.

$$(\ell+m+n+1)-\left\{\ell+m+\left[\frac{n+m-\ell}{2}\right]\right\}$$

$$=(n+1)-\left[\frac{n+m-\ell}{2}\right]$$

$$=\left[\frac{2n+2-n-m+\ell}{2}\right]$$

$$=\left[\frac{n-m+\ell+2}{2}\right]$$

$$=\left[\frac{n-m+\ell+2}{2}\right]$$

$$=\left[\frac{n-m+\ell}{2}\right]+1$$

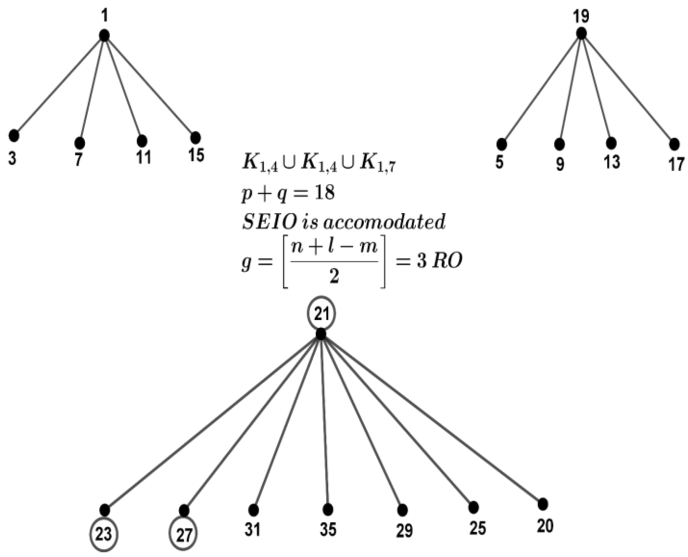
$$=g+1 \quad \text{where} \quad g=\left[\frac{n-m+\ell}{2}\right]$$

From claim (iii) and (iv), if $\ell+2m+4$ is odd then FEIO is accepted and if $\ell+3m+5$ is odd SEIO is accepted.

Case (i): when anyone FEIO or SEIO is accommodated, the number of even integers omitted is $(g+1)-1=g$ RO. so the graph is a g RO graph.

Case (ii): when both FEIO and SEIO are accommodated, the number of even integers omitted is $(g+1)-2=(g-1)$ RO. so the graph is a $(g-1)$ RO graph.

Case (iii): when both FEIO and SEIO are not accommodated, the number of even integers omitted is $(g+1)$ RO.so the graph is a $(g+1)$ RO graph.



Note

In the discussion of the above theorem we require two conditions on ℓ, m and n

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(i) $2 \le \ell$

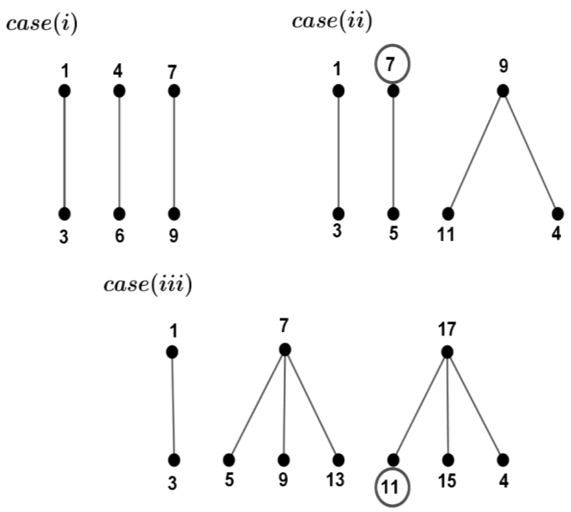
(ii) $2 \le (n-m)+\ell$

In particular when ℓ takes the value one, the following possibilities are to be considered.
In order to justify the conditions of the theorem, we must conclude the result of the theorem is not valid when $\ell = 1$ and $2 > (n-m)+\ell$.
By actual drawing, the three star graph $G = K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ for the values of ℓ, m and n mentioned below, we find the following:

(i) $\ell = m = n = 1$

(ii) $\ell = m = 1, n \ne 1$

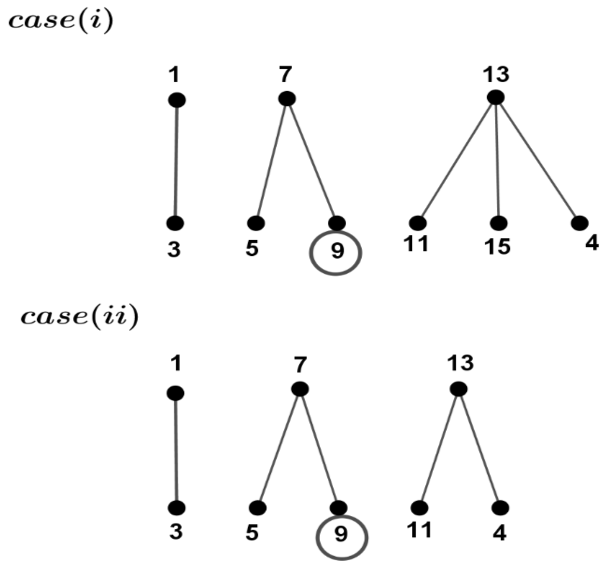
(iii) $\ell = 1, m = n =$ Odd integers



In these cases, we find both FEIO and SEIO can be accommodated but one of them alone is taken. While arriving at the value of g we had not come across this possibility. In these cases we cannot use g,
We now consider the following:

(i) $\ell = 1, m \ne n,$

(ii) $\ell = 1, m = n =$ even.



FEIO alone is accommodated and the graph is a g RO graph.
We observe that $\ell = 1$, we are not definite about the number of omissions and repetitions.
Hence the condition $2 \le \ell$, $2 \le (n-m)+\ell$, must hold in a general case.
In the above theorem, the nature of the SSMG graph, that is, $(g-1), (g)$ or $(g+1)$ RO graphs depends on the value of $\ell, m,$ and n where $g = \left\lceil \frac{n+\ell-m}{2} \right\rceil$.
The omissions and repetitions are found to be connected with FEIO and SEIO.
Here $\ell, m,$ and n are taken in general and they are not connected among themselves by any specific relations.

2.11: Discussion on omissions and repetitions when a specific relation exists among ℓ, m and n

When a specific relation exists among $\ell, m,$ and n, g is given in terms of n, after replacing ℓ and m in terms of n.
We now consider the case $\ell = m = n$ and find out the number of omissions and repetitions and

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express in terms of s where $n = 2s+1$ or $2s$, in the following theorem. We note that $g \neq s$ in general, when specific relations exist.

Theorem 2.12:

If $\ell = m = n$, the three star graph $G = K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ admits SSML (V_4, E_2) then it is a **s** RO graph for n odd or even where $n = 2s+1$ or $n = 2s$.

Proof

In the first copy of $G = K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ the labeling is done as follows:

$f(u) = 1,$

$f(u_1) = 3,$ $f(u_i) = 3 + 4(i-1),\ 2 \le i \le n$

$f(u_n) = 3 + 4(n-1) = 4n-1,$

The corresponding edge labeling

$f^*(uu_1) = 2,$

$f^*(uu_i) = 2i,\ 2 \le i \le n$

$f^*(uu_n) = 2n,$

Note that FEIO is $(2n+2)$

In the second copy of $G = K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ the labeling is done as follows:

Define

$f(v) = f(u_n) + 4 = (4n-1) + 4 = 4n+3$

$f(v_1) = 5,$

$f(v_j) = 5 + 4(j-1),\ 2 \le j \le n,$

$f(v_n) = 5 + 4(n-1) = 4n+1,$

The corresponding edge labeling

$f^*(vv_1) = 2n+4$

$$f^*(vv_j) = \frac{(4n+3) + 5 + 4(j-1)}{2}$$

$$= \frac{4n+4j+4}{2}$$

$$= 2n + 2j + 2, \quad 2 \le j \le n$$

$f^*(vv_n) = 4n+2$

Note that SEIO is $(4n+4)$

In the third copy of $G = K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ the labeling is done as follows:

Define

$f(w) = f(v_n) + 4 = (4n+1) + 4 = 4n+5$

$f(w_1) = 4n+7,$

$f(w_k) = (4n+7) + 4(k-1), \quad k = 1, 2, 3, \ldots K$

To obtain K:

$$4n + 3 + 4k \le 6n + 3$$

$$4k \le 2n$$

$$k \le \frac{n}{2}$$

$$K = \left[\frac{n}{2}\right]$$

So the number of even integers assigned up to the stage is

$$n + n + \left[\frac{n}{2}\right] = 2n + \left[\frac{n}{2}\right] \qquad \to (1)$$

Now a counting scheme is provided for the number of odd and even integers assigned and omitted.

$p + q = 6n + 3$ is an odd integer.

So, number of odd integers is one more than the number of even integers.

$p + q = (3n+1) + (3n+2)$

The total number of odd integers is $(3n+2)$ and the total number of even integers is (3n+1)

The even integers yet to be assigned

$$= (3n+1) - \left\{2n + \left[\frac{n}{2}\right]\right\}$$

$$= (n+1) - \left[\frac{n}{2}\right] \qquad \to (2)$$

The number of odd integers used up to the K^{th} position is given by

$(n+1) + (n+1) + 1 + K$

$= (2n+3) + K \quad \to (3)$

The number of odd integers yet to be assigned

$= (3n+2) - (2n+3+K)$

$= (n-1) - K$

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$= (n-1)-\left[\frac{n}{2}\right] \quad \rightarrow (4)$

The edge values that arise out of these odd integers and f (w) are also odd integers.

Because, the odd integers which are of difference four taken in order give even edge values.

The odd integers given in (4) must take care of $2(n-K)$ positions including the pendant vertices and edge values.

$2(n-K)-\left\{(n-1)-\left[\frac{n}{2}\right]\right\}$ are to be filled by even integers

$$2(n-K)-\left\{(n-1)-\left[\frac{n}{2}\right]\right\} =$$

$$2n-[n]-n+1+\left[\frac{n}{2}\right]$$

$$= n+1-\left[\frac{n}{2}\right]$$

$$= [n+1]-\left[\frac{n}{2}\right] \quad \rightarrow (5)$$

when n is odd, $[n+1]-\left[\frac{n}{2}\right]=\left[\frac{n+2}{2}\right]=\left[\frac{n+3}{2}\right] \quad \rightarrow (6)$

when n is even, $[n+1]-\left[\frac{n}{2}\right]=\left[\frac{n+2}{2}\right]=\left[\frac{n+2}{2}\right] \quad \rightarrow (7)$

Therefore when n is odd, the even numbers to be filled up = $\left[\frac{n+3}{2}\right]$ and

when n is even, the even numbers to be filled = $\left[\frac{n+2}{2}\right] \quad \rightarrow (8)$

Case (i): when n is odd

FEIO =2n+2,

By claim (iii), the condition for FEIO to be accommodated is $\ell+2m+4=3n+4$ is odd.

When n is odd, $3n+4$ is odd.

So, FEIO is accommodated when n is odd.

By claim (iv), the condition for SEIO to be accommodated is $\ell+3m+5=4n+5$ is odd.

When n is odd, $4n+5$ is odd.

So, SEIO is accommodated when n is odd.

Hence the number of even integers omitted = $\left[\frac{n+3}{2}\right]-2$

When n is odd, $n=2s+1, \left[\frac{2s+1+3}{2}\right]-2=(s+2)-2=s$

Hence when n is odd, the graph $G=K_{1,\ell}\cup K_{1,m}\cup K_{1,n}$ is a s RO graph.

Case (ii): when n is even

FEIO = 2n+2,

By claim (iii), the condition for FEIO to be accommodated is $\ell+2m+4=3n+4$ is odd.

When n is even, $3n+4$ is even.

So, FEIO cannot be accommodated when n is even.

By claim (iv), the condition for SEIO to be accommodated is $\ell+3m+5=4n+5$ is odd.

When n is even, $4n+5$ is odd.

So, SEIO is accommodated when n is even.

Hence the number of even integers omitted = $\left[\frac{n+2}{2}\right]-1$

When n is even, $n=2s, \left[\frac{2s+2}{2}\right]-1=(s+1)-1=s$

Hence when n is even, the graph $G=K_{1,\ell}\cup K_{1,m}\cup K_{1,n}$ is a s RO graph.

The following table shows the nature of the graph in terms of g and s using the theorems 2.10 and 2.11.

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Table 1: Relation between g and s on $G = K_{1,\ell} \cup K_{1,m} \cup K_{1,n},\ 2 \le \ell,\ 2 \le (n-m)+\ell.$

Specific relations among ℓ, m and n		**FEIO**	**SEIO**	**Interms of** $g=\left[\frac{n+\ell-m}{2}\right]$	**Interms of s where n=2s+1 or 2s**	**Particular values**
ℓ, m and n are equal $\ell = m = n$	ℓ, m and n odd	✓	✓	$(g-1)$ RO	s RO	ℓ =m=n=5 2 RO
ℓ, m and n are equal $\ell = m = n$	ℓ, m and n even	✗	✓	g RO	s RO	ℓ =m=n=6 3 RO
ℓ, m and n are consecutive integers $\ell < m < n$	ℓ and n odd m even	✓	✗	g RO	s RO	ℓ =5,m=6,n=7 3 RO
ℓ, m and n are consecutive integers $\ell < m < n$	m odd ℓ and n even	✗	✗	$(g+1)$ RO	$(s+1)$ RO	ℓ =6,m=7,n=8 5 RO
ℓ, m and n are consecutive odd integers $\ell < m < n$	ℓ, m and n odd	✓	✓	$(g-1)$ RO	$(s-1)$ RO	ℓ =5,m=7,n=9 3 RO
ℓ, m and n are consecutive even integers $\ell < m < n$	ℓ, m and n even	✗	✓	g RO	$(s-1)$ RO	ℓ =8,m=10,n=12 5 RO
ℓ and m are equal, n different $\ell = m < n$	ℓ and m odd n even	✓	✓	$(g-1)$ RO	$(s-1)$ RO	ℓ =11,m=11,n=14 6 RO
ℓ and m are equal, n different $\ell = m < n$	ℓ, m and n odd	✓	✓	$(g-1)$ RO	s RO	ℓ =11,m=11,n=17 8 RO

ℓ and m are equal, n different $\ell = m < n$	ℓ and m even n odd	✕	✓	g RO	(s+1) RO	ℓ=6,m=6,n=9 5 RO
ℓ and m are equal, n different $\ell = m < n$	ℓ, m and n even	✕	✓	g RO	s RO	ℓ=8,m=8,n=10 5RO
The above table shows the nature of the SSMG given in terms of g in general and s when *l*,m and n share a specific relation. Here $g \neq s$ in general. The two approaches when *l*,m and n are taken in general and when they are connected by a relation, dealt with in theorems 2.9 and theorem 2.11 respectively lead to different forms in solutions is good to note.						

Conclusion

We have proved that any three star graph $G = K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$, $\ell \le m \le n$ for various values of ℓ, m and n with $2 \le (n-m)+\ell, 2 \le \ell$, is $(g-1),(g)$ and $(g+1)$ RO sub super mean graphs where $g = \left\lceil \frac{n+\ell-m}{2} \right\rceil$ according as both FEIO and SEIO are accommodated, anyone of FEIO and SEIO is accommodated and both FEIO and SEIO not accommodated respectively.

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