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Study the Effects of Mass in a Simple Damped Harmonic Motion of a Mass-spring System Using Simulation

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Abstract

The effect of mass on the behavior of oscillatory systems in a damped spring-mass system was studied using simulation. It was found that the mass affects the amplitude and displacement in the case of an undamped oscillatory system. In critically damped systems, the mass affects the displacement exponentially, and the system doesn’t oscillate. In the case of an overdamped system, there is also no oscillatory motion, and an increase in the mass was not affected, since the system gets to rest very quickly. The study shows that simulation can be a very helpful tool to study the behavior of oscillatory physical systems.
Keywords: Simple harmonic motion damped mass-Spring system, simulation

Introduction

When the motion of an oscillator reduces due to an external force, the oscillator and its motion are damped. These periodic motions of gradually decreasing amplitude are damped simple harmonic motion. In the damped harmonic motion, the energy of the oscillator dissipates continuously. But for small damping, the oscillations remain approximately periodic. The forces which dissipate the energy are generally frictional forces. Many researchers made studies on simple harmonic oscillators experimentally [1-3], while others studied these oscillations theoretically [4-11]. The aim of this work was designed to investigate the effect of mass on the damping behavior of a simple harmonic oscillator using the simulation. The oscillating system is chosen to be a mass-spring system rather than a pendulum. The damped system depends on the damping coefficient, which in turn depends on the value of the mass, spring constant, and damping ratio. A damping ratio is a dimensionless number bigger than zero that depends on the state of the oscillating system, whether it is underdamped, over-damped, or critically damped.

Theory

The equation of motion of a simple damped harmonic oscillator is given by the following expression

$$m\frac{\partial^2}{\partial t^2}\,x(t)+c\,\frac{\partial}{\partial t}x(t)+k\,x(t)=F(t)$$

Using $c = m\gamma$ and $k = m\omega_0^2$, the above equation can be rewritten as

$$m\frac{\partial^2}{\partial t^2}\,x(t)+\gamma\,m\frac{\partial}{\partial t}x(t)+m\,x(t)\omega_0^2=F(t)$$

Divide out the mass *m*. Now results have the equation in a convenient form to analyze.

$$\frac{\partial^2}{\partial t^2}\,x(t)+\gamma\,\frac{\partial}{\partial t}x(t)+x(t)\omega_0^2=\frac{F(t)}{m}$$

In case of: F = 0, the equation can be written as

$$sol=\frac{e^{-t\left(\frac{\gamma}{2}-\frac{\sigma_1}{2}\right)}(\gamma+\sigma_1)}{2\sigma_1}-\frac{e^{-t\left(\frac{\gamma}{2}+\frac{\sigma_1}{2}\right)}(\gamma-\sigma_1)}{2\sigma_1},\,where\,\sigma_1=\sqrt{(\gamma-2\omega_0)(\gamma+2\omega_0)}$$

This can be simplified as

$$sol=\frac{\sigma_1\sigma_2}{2}+\frac{\sigma_1\,e^{\frac{t\sqrt{\gamma^2-4\omega_0^2}}{2}}}{2}-\frac{\gamma\,\sigma_1\sigma_2}{2\sqrt{\gamma^2-4\omega_0^2}}+\frac{\gamma\,\sigma_1\,e^{\frac{t\sqrt{\gamma^2-4\omega_0^2}}{2}}}{2\sqrt{\gamma^2-4\omega_0^2}}$$

$$where\,\sigma_1=e^{-\frac{\gamma t}{2}},\sigma_2=e^{-\frac{t\sqrt{\gamma^2-4\omega_0^2}}{2}}$$

Here, it was noticed that each term has a factor of σ_1, or $e^{\frac{-\gamma t}{2}}$

That can gather these terms to get the following expression

$$sol=\left(\frac{e^{-\sigma_1}}{2}+\frac{e^{\sigma_1}}{2}-\frac{\gamma\,e^{-\sigma_1}}{2\sqrt{\gamma^2-4\omega_0^2}}+\frac{\gamma\,e^{\sigma_1}}{2\sqrt{\gamma^2-4\omega_0^2}}\right)\mathrm{e}^{-\frac{\gamma t}{2}}$$

$$where\,\sigma_1=\frac{t\sqrt{\gamma^2-4\omega_0^2}}{2}$$

The term $\sqrt{\gamma^2 - 4\omega_0^2}$ appears in various parts of the solution. It could be written in a simpler form by introducing the damping ratio $\zeta \equiv \frac{\gamma}{2\omega_0}$.

Substituting ζ into the term above gives:

$$\sqrt{\gamma^2 - 4\omega_0^2} = 2\omega_0 \sqrt{\left(\frac{\gamma}{2\omega_0}\right)^2 - 1} = 2\omega_0 \sqrt{\zeta^2 - 1},$$

$$sol = e^{-\frac{\gamma t}{2}} \left(\frac{\sigma_2}{2} + \frac{\sigma_1}{2} + \frac{\gamma\, \sigma_2}{4\omega_0 \sqrt{\zeta^2 - 1}} - \frac{\gamma\, \sigma_1}{4\omega_0 \sqrt{\zeta^2 - 1}} \right)$$

$$where\, \sigma_1 = e^{-\omega_0 t \sqrt{\zeta^2 - 1}}, \sigma_2 = e^{\omega_0 t \sqrt{\zeta^2 - 1}}$$

Further, that might simplify the solution by substituting γ in terms of ω_0 and ζ ,

$$sol = e^{-\omega_0 t\, \zeta} \left(\frac{\sigma_2}{2} + \frac{\sigma_1}{2} + \frac{\zeta\, \sigma_2}{2\sqrt{\zeta^2 - 1}} - \frac{\zeta\, \sigma_1}{2\sqrt{\zeta^2 - 1}} \right)$$

$$where\, \sigma_1 = e^{-\omega_0\, t \sqrt{\zeta^2 - 1}}, \sigma_2 = e^{\omega_0\, t \sqrt{\zeta^2 - 1}}$$

Under damped Case ($0<\zeta<1$**)**

For an underdamped system, the damping ratio is between zero and one. This is the most common case and the only one that yields oscillation.

If $0 < \zeta < 1$, then $\sqrt{\zeta^2 - 1} = i\sqrt{1 - \zeta^2}$ is purely imaginary

$$solunder = e^{-\omega_0 t \zeta} \left(\frac{\sigma_1}{2} + \frac{\sigma_2}{2} + \frac{\zeta\, \sigma_1 i}{2\sqrt{1 - \zeta^2}} - \frac{\zeta\, \sigma_2\, i}{2\sqrt{1 - \zeta^2}} \right)$$

$$where\, \sigma_1 = e^{-\omega_0 t \sqrt{1 - \zeta^2}\, i}, \sigma_2 = e^{\omega_0 t \sqrt{1 - \zeta^2}\, i}$$

Here it was noticed the terms $e^{i\omega_0 t \sqrt{\zeta^2 - 1}} \pm e^{-i\omega_0 t \sqrt{\zeta^2 - 1}}$ appearing in the above equation and by using the trigonometric identity, $e^{ix} = \cos(x) + i\sin(x)$, it could be rewritten the solution in terms of cos.

$$solunder = e^{-\omega_0 t \zeta} \cos\left(\omega_0 t \sqrt{1 - \zeta^2}\right)$$

$$solunder(t, omega\_\theta, zeta) = e^{-\omega_0 t \zeta} \cos\left(\omega_0 t \sqrt{1 - \zeta^2}\right)$$

The system oscillates at a natural frequency of $\omega_0\sqrt{1-\zeta^2}$ and decays at an exponential rate of $1/\omega_0\zeta$.

This type of system gives an oscillation response with exponential decay. Most of the natural systems oscillate in this way. The underdamped oscillation has its own frequency of oscillation called the "damping frequency".

Figure 1, shows the oscillation of a theoretical underdamped system.

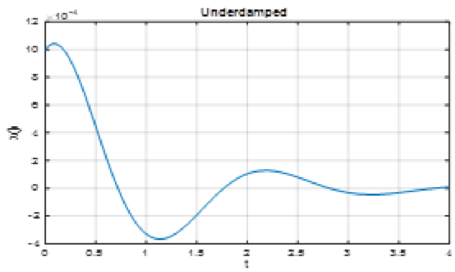


Figure 1: The theoretical underdamped case.

Over damped Case (ζ>1)

In an over-damped system, the damping ratio is greater than 1.

If $\zeta > 1$, then $\sqrt{\zeta^2 - 1}$ is purely real and the solution can be rewritten as

$$solover = e^{-\omega_0 t\zeta}\left(\frac{\sigma_2}{2}+\frac{\sigma_1}{2}+\frac{\zeta\,\sigma_2}{2\sqrt{\zeta^2-1}}-\frac{\zeta\,\sigma_1}{2\sqrt{\zeta^2-1}}\right)$$

$$where\,\sigma_1 = e^{-\omega_0 t\sqrt{\zeta^2-1}},\sigma_2 = e^{\omega_0 t\sqrt{\zeta^2-1}}$$

$$solover = e^{-\omega_0 t\zeta}\left(\frac{e^{\omega_0 t\sqrt{\zeta^2-1}}}{2}+\frac{e^{-\omega_0 t\sqrt{\zeta^2-1}}}{2}\right)$$

Notice the terms $\dfrac{\left(e^{\omega_0 t\sqrt{\zeta^2-1}}+e^{-\omega_0 t\sqrt{\zeta^2-1}}\right)}{2}$ and recall the

identity $\cosh(x) = \dfrac{e^x + e^{-x}}{2}$.

Rewrite the expression in terms of cosh.

$$solover = \cosh\left(\omega_0 t\sqrt{\zeta^2-1}\right)e^{-\omega_0 t\,\zeta}$$

Figure 2, shows the oscillation of a theoretical overdamped system.

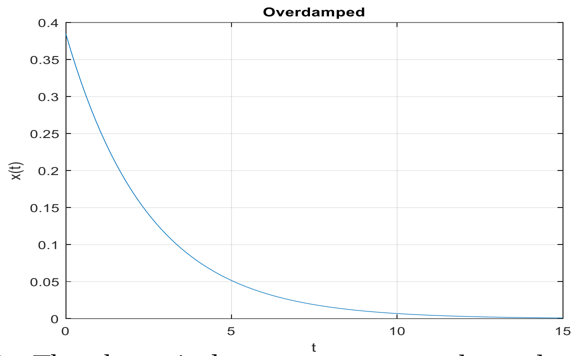


Figure 2: The theoretical overdamped case

Critically Damped Case (ζ=1)

For a critically damped system, the value of the damping ratio is equal to 1. In this case, also no oscillation occurs.

If ζ=1, then the solution simplifies to $solcritical(t, omega\_\theta) = e^{-\omega_0 t}(\omega_0 t + 1)$

Figure 3, shows the critically damped oscillation.

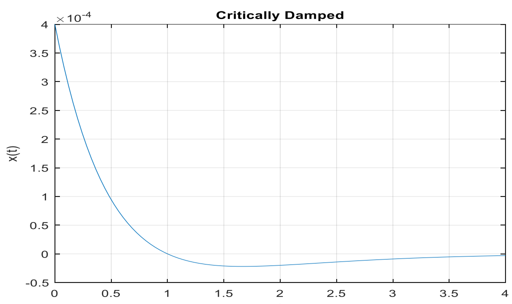


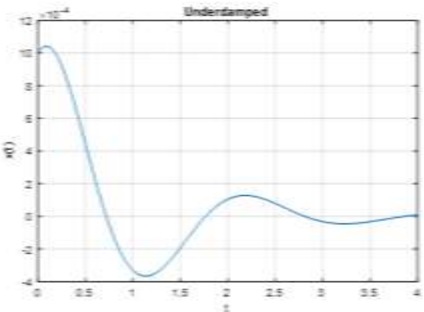
Figure 3: The theoretical critically damped case

Results and Discussion

The effect of varying the mass on the oscillatory behavior is discussed for the three cases.

The underdamped case

In this case, to study the effect of mass on the behavior of the oscillating system, the mass was varied from 1 to 4 kgs, while the spring constant was held at 10 N/m and the damping ratio at 2.
Figures 4, 5, 6, and 7were showed the effect of mass. It can be seen that there is an oscillating behavior for this type of damping and that as **m** becomes larger and larger, the amplitude decreases with time and there is a decrease in the displacement as a function of time.



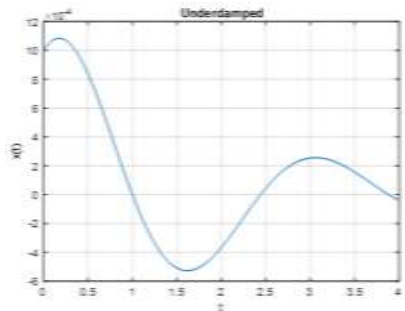
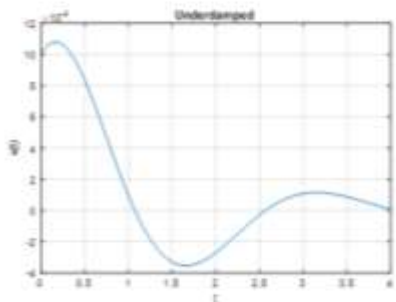


Figure 4: m=1kg, k=10 N/m, c=2 **Figure 5:** m=2kg, k=10N/m, c=2



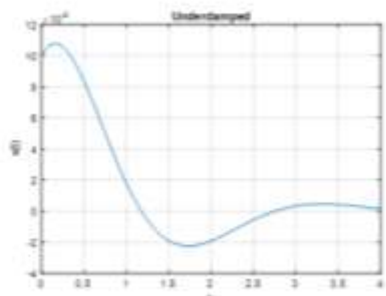
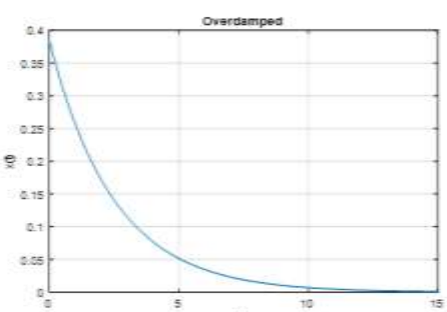


Figure 6: m=3kg,k=10N/m,c=2 **Figure7:** m=4kg, k=10N/m, c=2

The overdamped case

In this case, to study the effect of mass on the behavior of the oscillating system, the mass was varied from 100kgs to 250 kgs, while the spring constant was held at 225 N/m and the damping ratio at 600.
Figures 8, 9, 10, and 11 were shown the effect of mass in this case. The main feature is that the system doesn't oscillate at all, and there is no effect of increasing the mass because it is already overdamped and came to a complete rest.



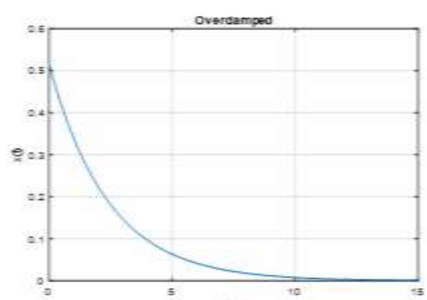
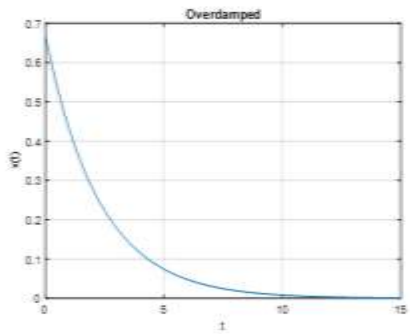


Figure 8: m=100kg,k=225N/m,c=600 **Figure 9:** m=150kg, k=225N/m, c=600



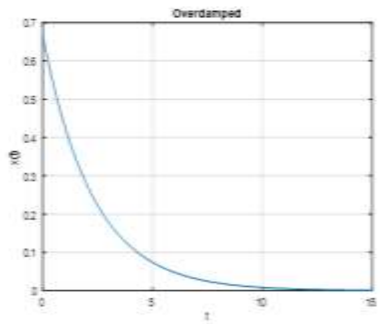
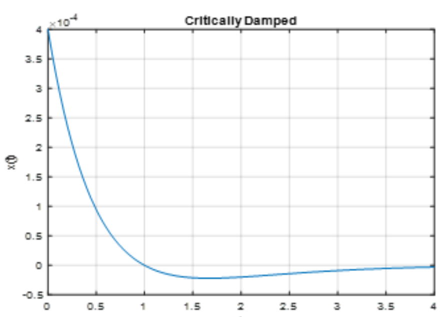


Figure 10: m=200kg, k=225N/m, c=600 **Figure 11:** m=250kg, k=225N/m, c=600

The critically damped case

In this case, to study the effect of mass on the behavior of the oscillating system for this type of damping, the mass was also varied from 100kgs to 250 kgs, while the spring constant was held at 225 N/m and the damping ration at 300.

Figures 12, 13, 14, and 15 were shown the effect of mass in this case. The main feature is that the system also doesn't oscillate and remains in its rest position, but as the mass increases, the displacement decreases more sharply, also in an exponential manner.



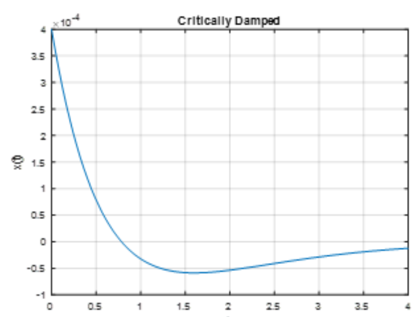
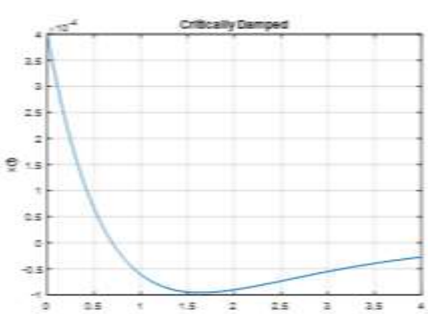


Figure12: M=100kg,K= 225N/m, c= 300 **Figure13:** M= 150 kg, K= 225N/m, c= 300



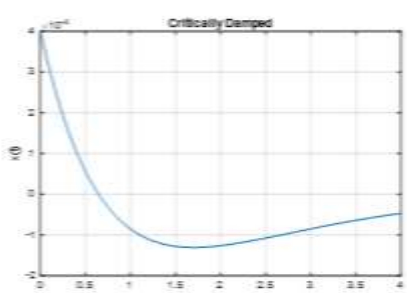


Figure 14: m=200 kg, K=225N/m,c= 300 **Figure 15:** m= 250 kg, K= 225N/m, c= 300

Conclusion

It can be concluded from this study that there is an effect of increasing the mass on the behavior of oscillation in the underdamped case, as the amplitude decreases with time by increasing the mass, as well as the increase of mass affects the displacement inversely. In the case of overdamping, no effect is seen, while for the critically damped case, there is no oscillation, but the displacement decreases exponentially and sharply. This study shows that the tool of simulation is very helpful in studying the behavior of physical oscillatory systems in the case of simple harmonic damped systems.

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